

# Status of polarized pdf's and the role of the gluon

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Electromagnetic Interactions with Nucleons and Nuclei

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# Outline

- Introduction: spin sum rule
- DSSV analysis
- The polarized Gluon
- Summary

# Ουτλινε

- Ιντροδυχτιον : σπιν συμ ρυλε
- ΔΣΣς αναλψσις
- Τηε πολαριζεδ Γλυον
- Συμμαρψ

# (Longitudinal) Spin structure of the proton

Spin share between components

$$\frac{1}{2} \underbrace{\Delta \Sigma}_{\text{Quark Spin}} + \underbrace{\Delta G}_{\text{Gluon Spin}} + \underbrace{L_q + L_G}_{\text{OAM}} = \frac{1}{2}$$

Jaffe-Manohar

Naive expectation SU(6) : 70% corresponds to Quark Spin

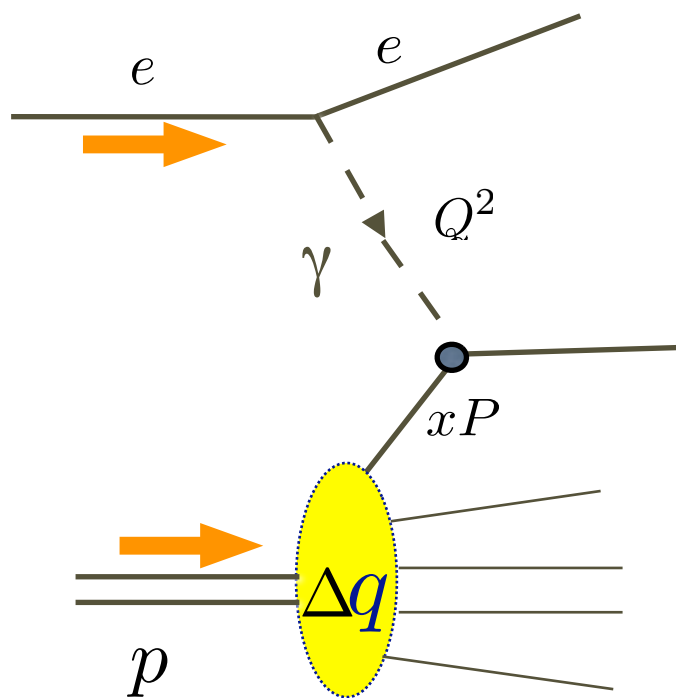
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Measure at polarized DIS : structure function



$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)]$$

Polarized PDFs  $\Delta f_i(x, Q^2) \equiv f_i^\uparrow(x, Q^2) - f_i^\downarrow(x, Q^2)$

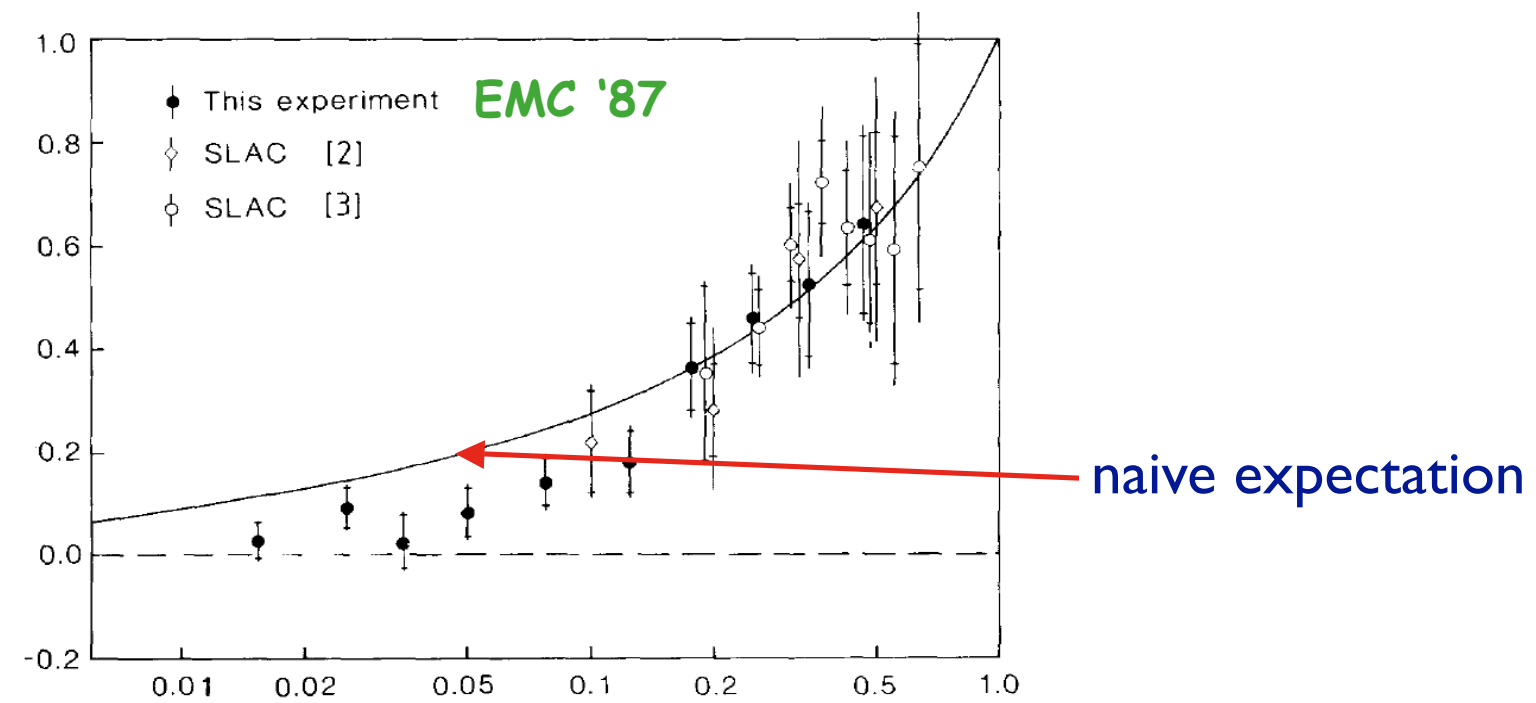
$$\frac{d}{d \ln Q^2} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} (x, Q^2) = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta P_{qq}(\alpha_s(Q^2), z) & \Delta P_{qg}(\alpha_s(Q^2), z) \\ \Delta P_{gq}(\alpha_s(Q^2), z) & \Delta P_{gg}(\alpha_s(Q^2), z) \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} \left( \frac{x}{z}, Q^2 \right)$$

Quark and Gluon spin contributions

$$\Delta\Sigma = \sum_i \int_0^1 \Delta q_i(x, Q^2) dx \quad \Delta G = \int_0^1 \Delta g(x, Q^2) dx$$

# 1987 : EMC

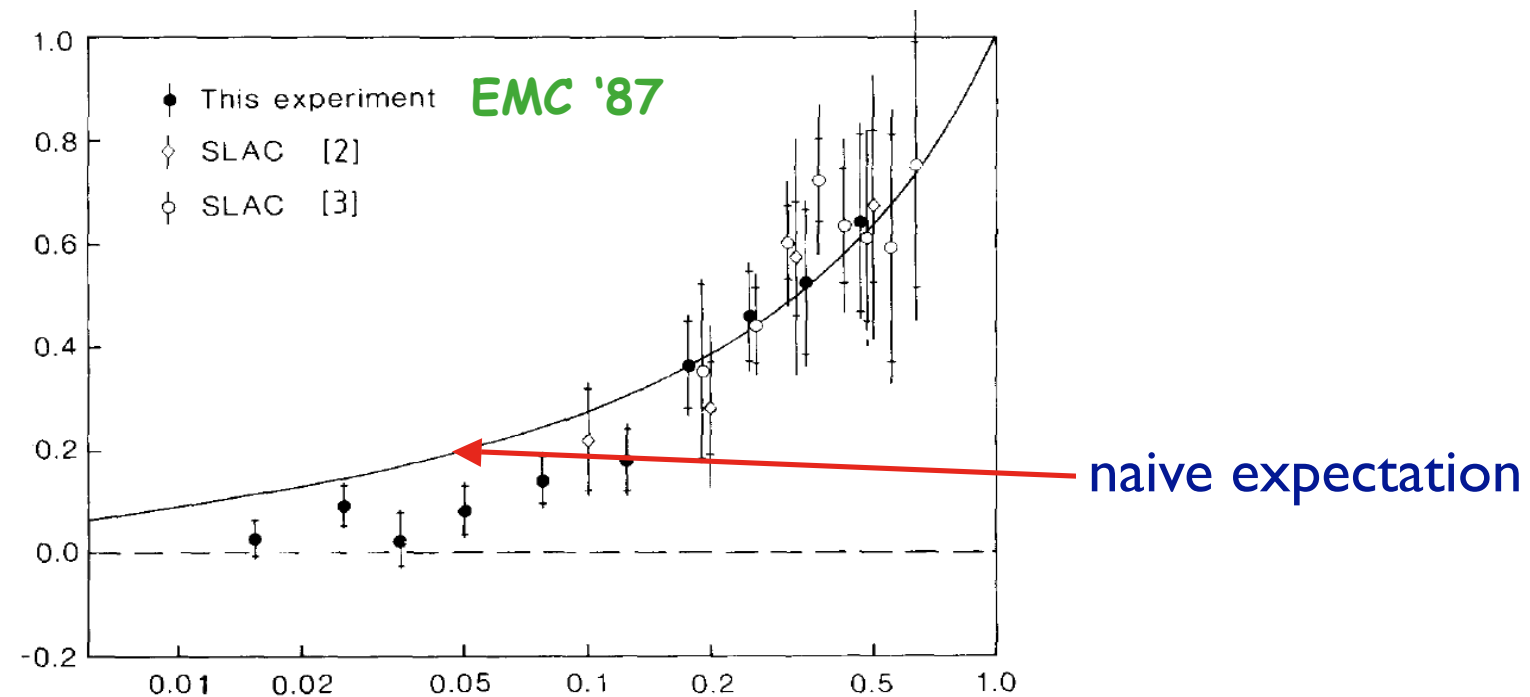
$$A_1(x) = \frac{g_1(x)}{F_1(x)}$$



$\Delta\Sigma \sim 0.0 \pm 0.24 \quad (Q^2 = 10 \text{ GeV}^2) \quad \text{'spin-crisis puzzle'}$

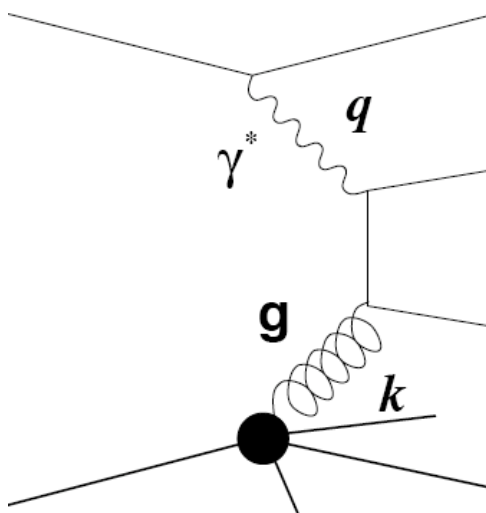
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Any way to recover naive expectation?



Gluon contribution through Anomaly

Altarelli, Ross  
Efremov, Teryaev

'gluon contamination'

$$\Delta\Sigma^{exp} = \Delta\Sigma - 3 \frac{\alpha_s}{2\pi} \Delta G$$

~0

~0.7

$\Delta G > 5 !!$

unnaturally large

Increased the interest on measuring the gluon contribution (full set of pdfs)

# Polarized PDFs

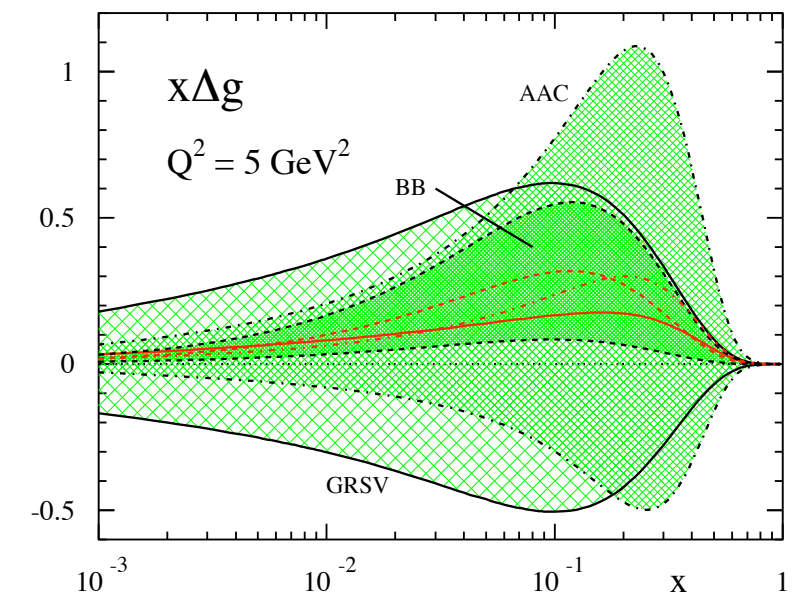
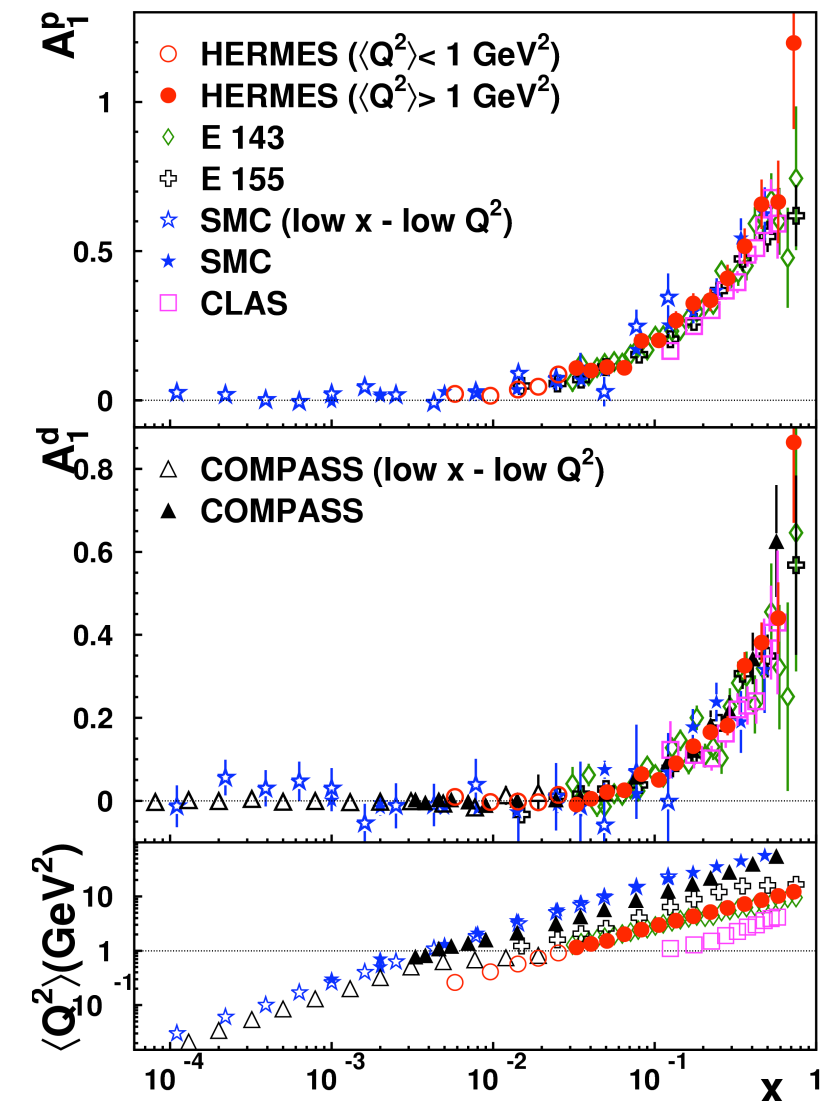
Several attempts to obtain polarized PDFs  
during the last decade

A few of them:

- E. Leader, A.V. Sidorov and D.B. Stamenov, **LSS**
- M. Glück, E. Reya, M. Stratmann and W. Vogelsang, **GRSV**
- T. Gehrmann and W.J. Stirling, **GS**
- J. Bluemlein and H. Boettcher, **BB**
- Asymmetry Analysis Collaboration, **AAC**
- DdF and R. Sassot, **DS**
- DdF, G.A. Navarro and R. Sassot, **DNS**
- C. Bourrely, F. Buccella and J. Soffer, **BBS**
- G. Altarelli, R. Ball, S. Forte, G. Ridolfi, **ABFS**
- **SMC**
- **HERMES**
- + several others

DIS fixed target

$$\Delta G(Q^2 = 10) \sim 1 - 2$$



Aidala et al

All fits include DIS data : agree on 'quark'  
Differ on assumptions about the sea  
Huge differences on gluon distribution



experiment	data type	data points fitted
EMC, SMC	DIS	34
COMPASS	DIS	15
E142, E143, E154, E155	DIS	123
HERMES	DIS	39
HALL-A	DIS	3
CLAS	DIS	20
SMC	SIDIS, $h^\pm$	48
HERMES	SIDIS, $h^\pm$	54
	SIDIS, $\pi^\pm$	36
	SIDIS, $K^\pm$	27
COMPASS	SIDIS, $h^\pm$	24
PHENIX (in part prel.)	200 GeV pp, $\pi^0$	20
PHENIX (prel.)	62 GeV pp, $\pi^0$	5
STAR (in part prel.)	200 GeV pp, jet	19
TOTAL:		467

$\Delta q + \Delta \bar{q}$

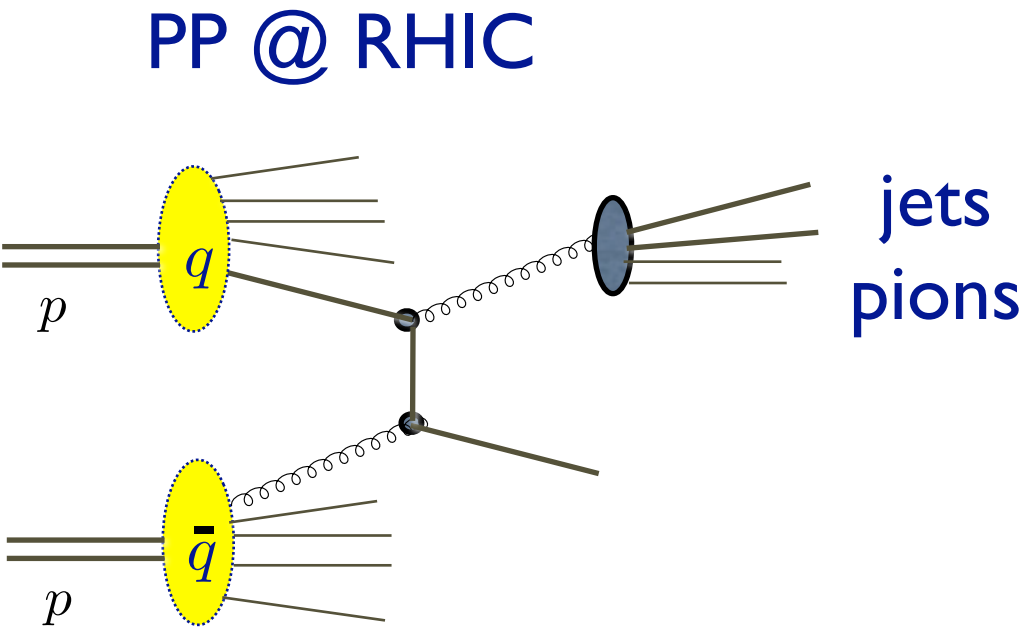
$\Delta q, \Delta \bar{q}$

$\Delta g$

jet @ STAR  $p_T \geq 5 \text{ GeV}$

pions @ Phenix  $p_T^\pi \geq 1 \text{ GeV}$

DIS/SIDIS  $Q \geq 1 \text{ GeV}$



Gluons at LO !

$x \sim 0.05 - 0.2$

# Data Selection

Important lesson from unpolarized physics : use “safe” observables

Parton Model Factorization

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \overset{\text{pdfs}}{\Delta f_a(x_a, Q^2) \Delta f_b(x_b, Q^2)} \times \underset{\text{partonic cross-section}}{d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)}$$

Only Rule (not bias): Check that works on the unpolarized case!

Solid basis for the analysis

First develop new set of fragmentation functions to

validate semi-inclusive processes

**DSS fragmentation** (DdeF, Sassot, Stratmann)

from e<sup>+</sup>e<sup>-</sup>, ep and pp collisions

**SIDIS** ✓      $pp \rightarrow \pi$  ✓

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Do not include: “high p<sub>T</sub>” hadrons from Compass and Hermes

No NLO description available (soon!)

Not clear pQCD works well : check unpolarized?

# Global Fit

$$x\Delta f_j(x, Q_0^2) = N_j x^{\alpha_j} (1-x)^{\beta_j} (1 + \gamma_j \sqrt{x} + \eta_j x)$$

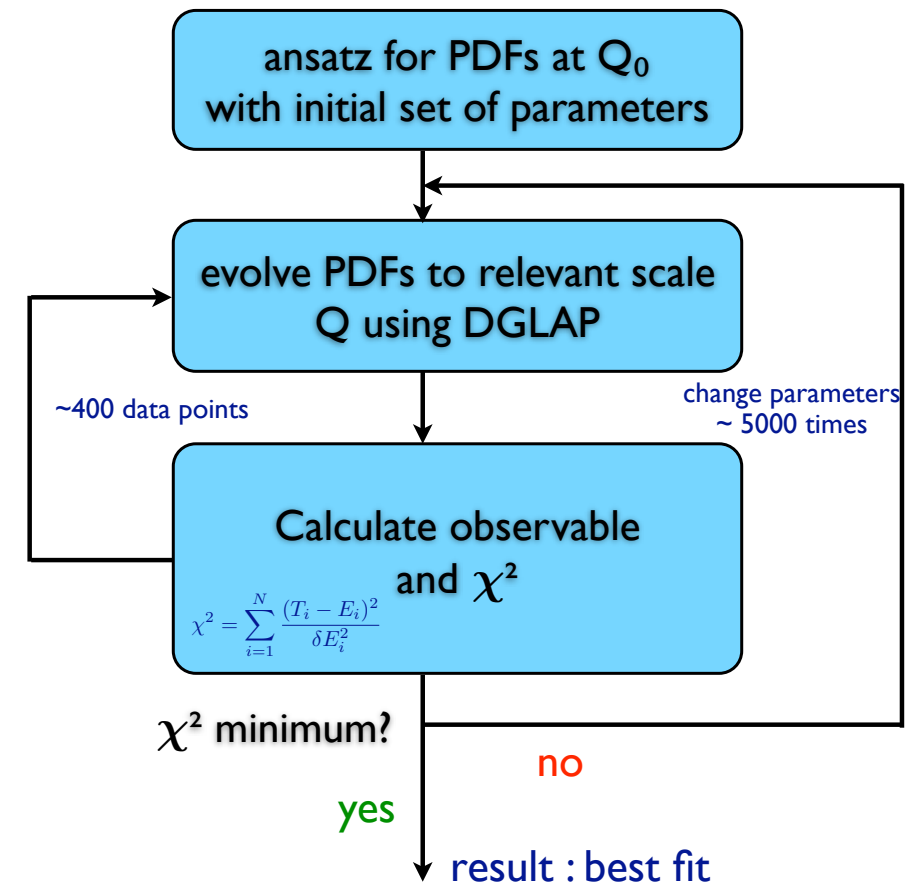
0 for sea/gluon

node allowed

Some constraints from unpol./sym.

21 free parameters

PDFs obtained by global fit :  $\chi^2$  minimization



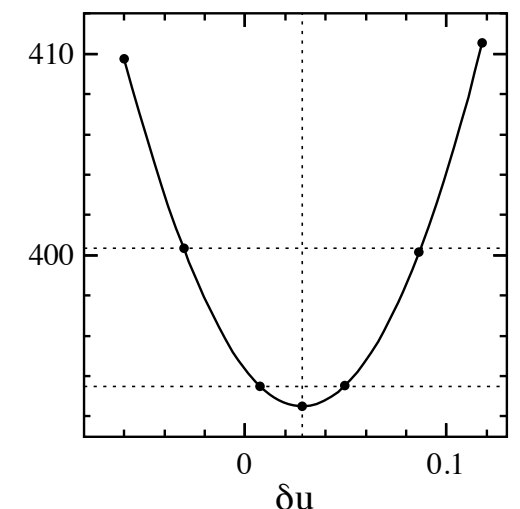
Use **Lagrange Multipliers** technique to **estimate uncertainties** (from exp. errors) on **some observables**

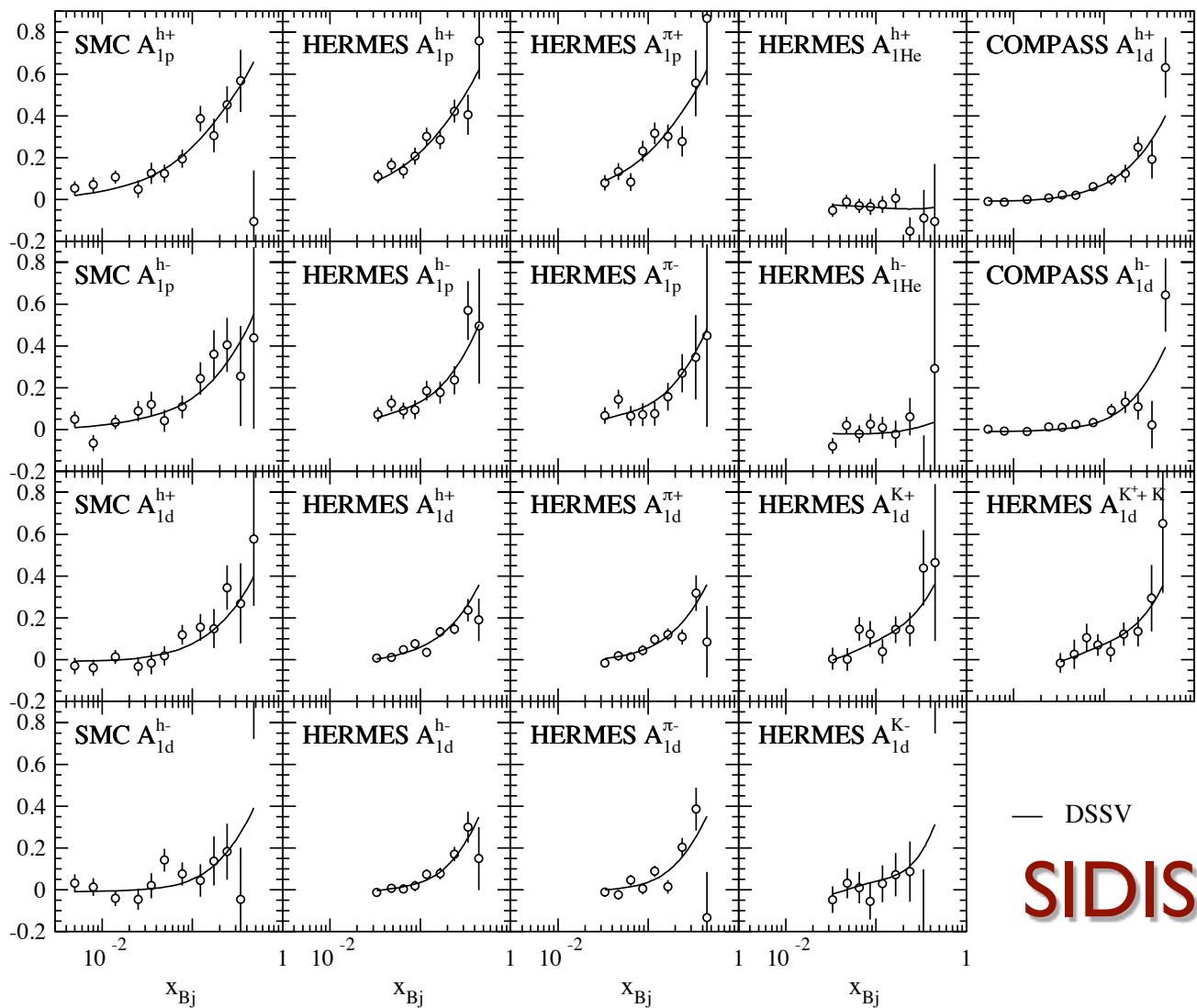
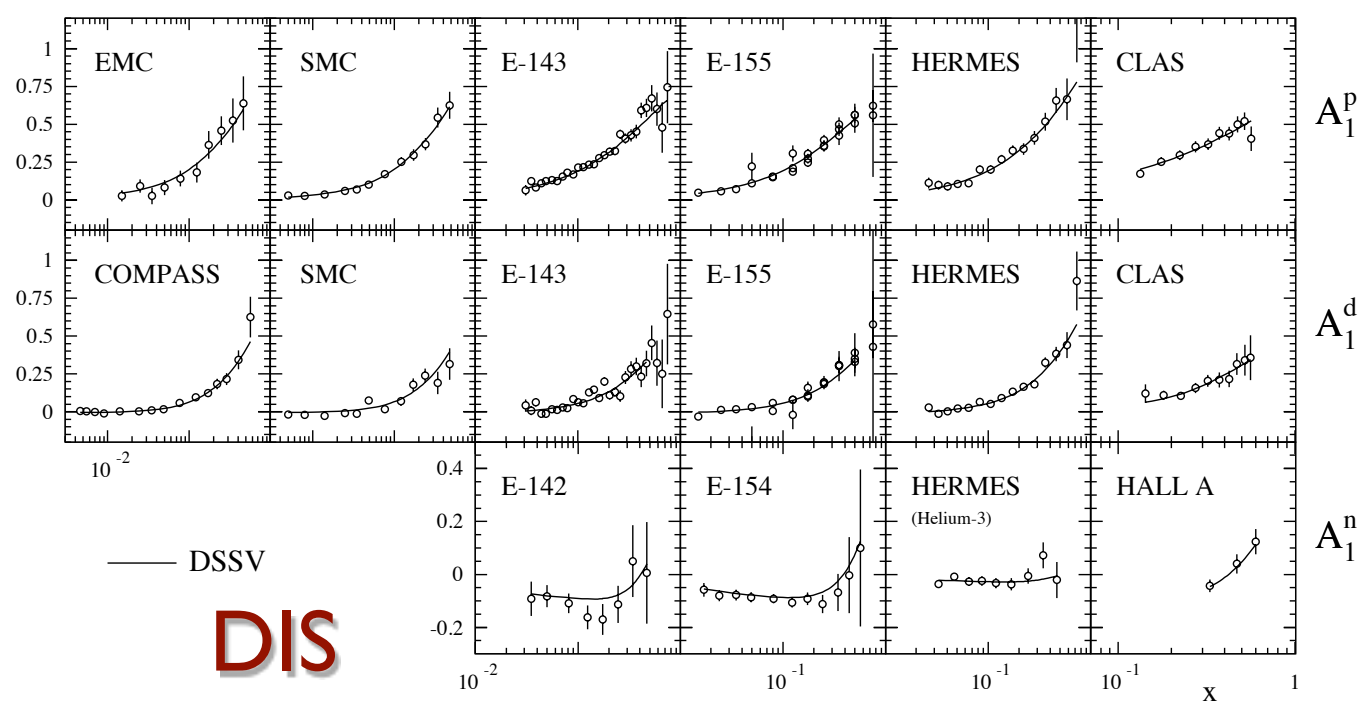
$$\Phi(\lambda_i, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i \mathcal{O}_i(\{a_j\})$$

See how fit deteriorates when PDFs forced to give different prediction for  $\mathcal{O}_i$

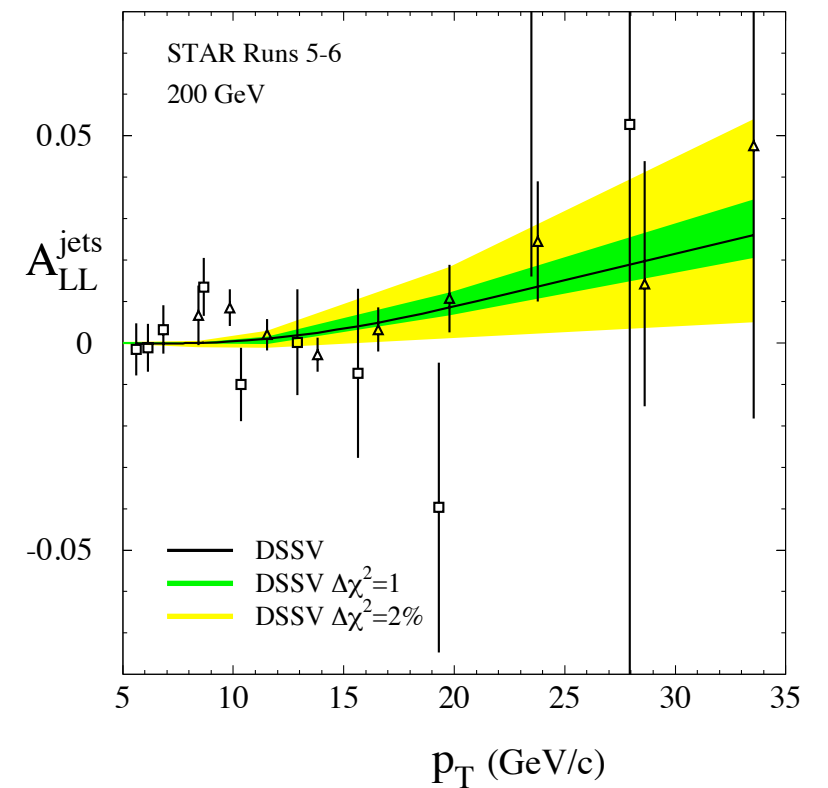
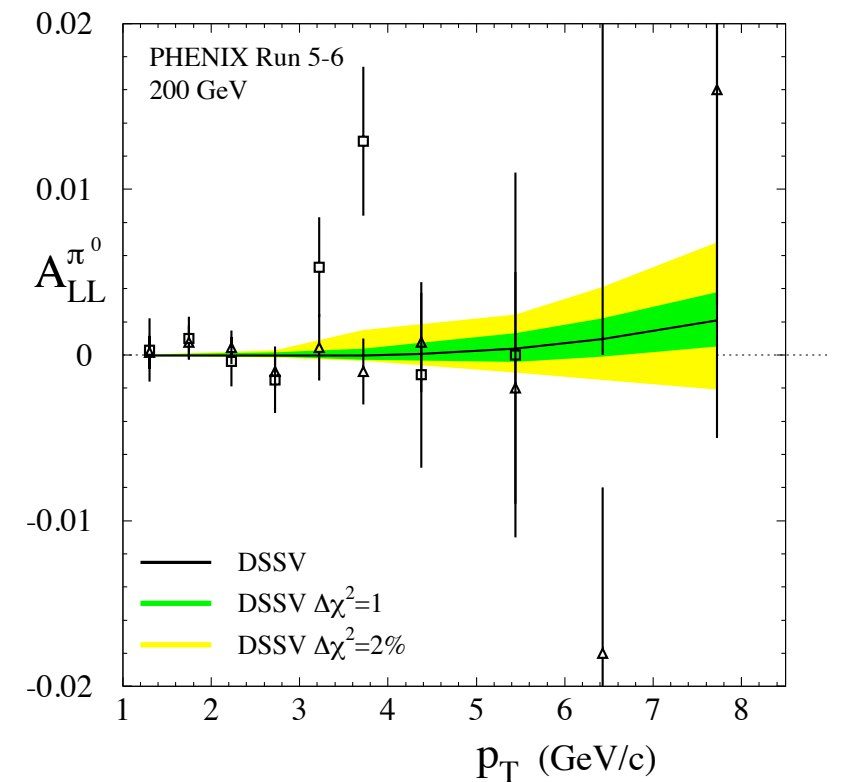
$\Delta\chi_n^2$  We take a pragmatic 2% to define **uncertainty bands**

First moments as 'observables'





# RHIC observables

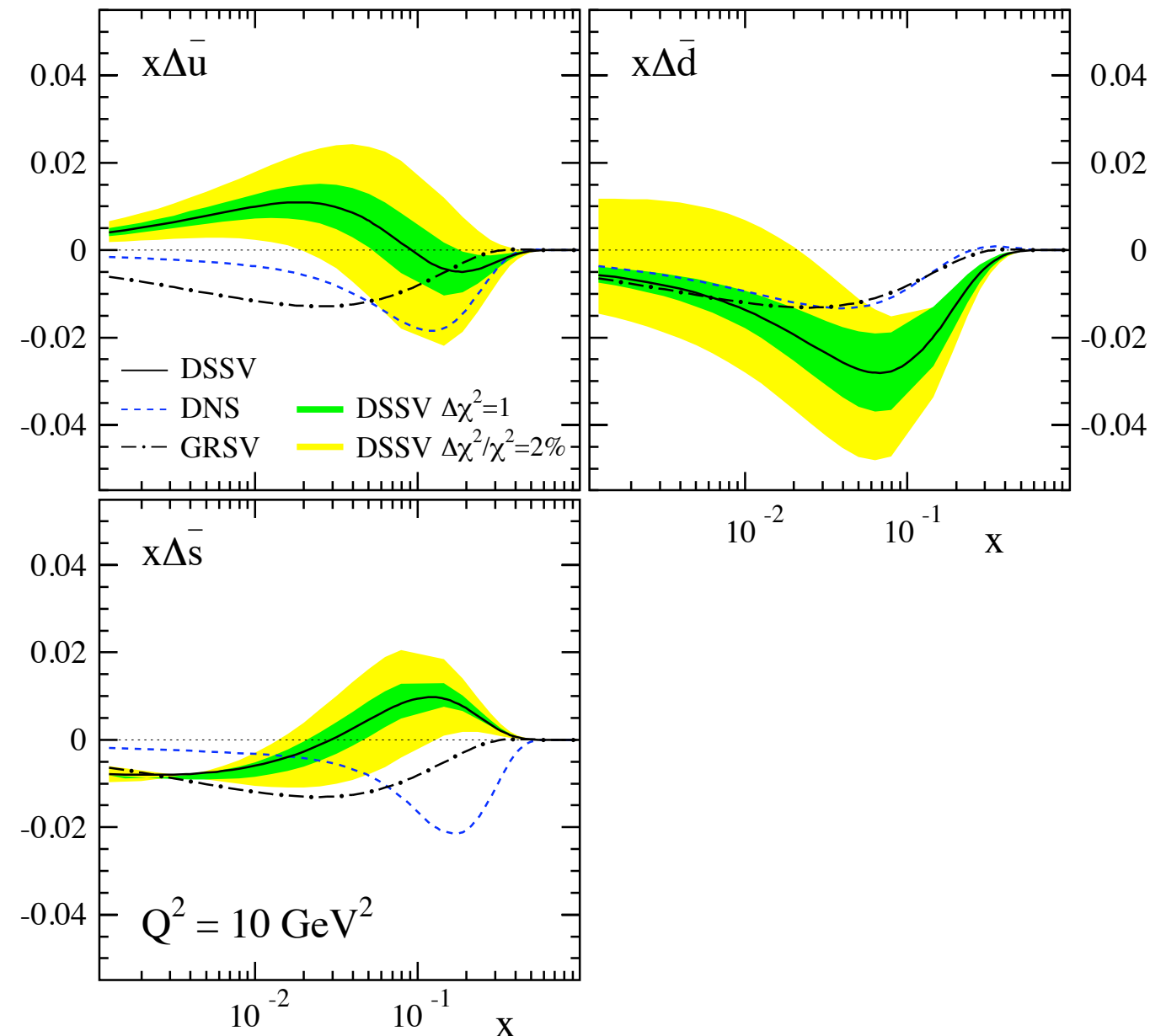
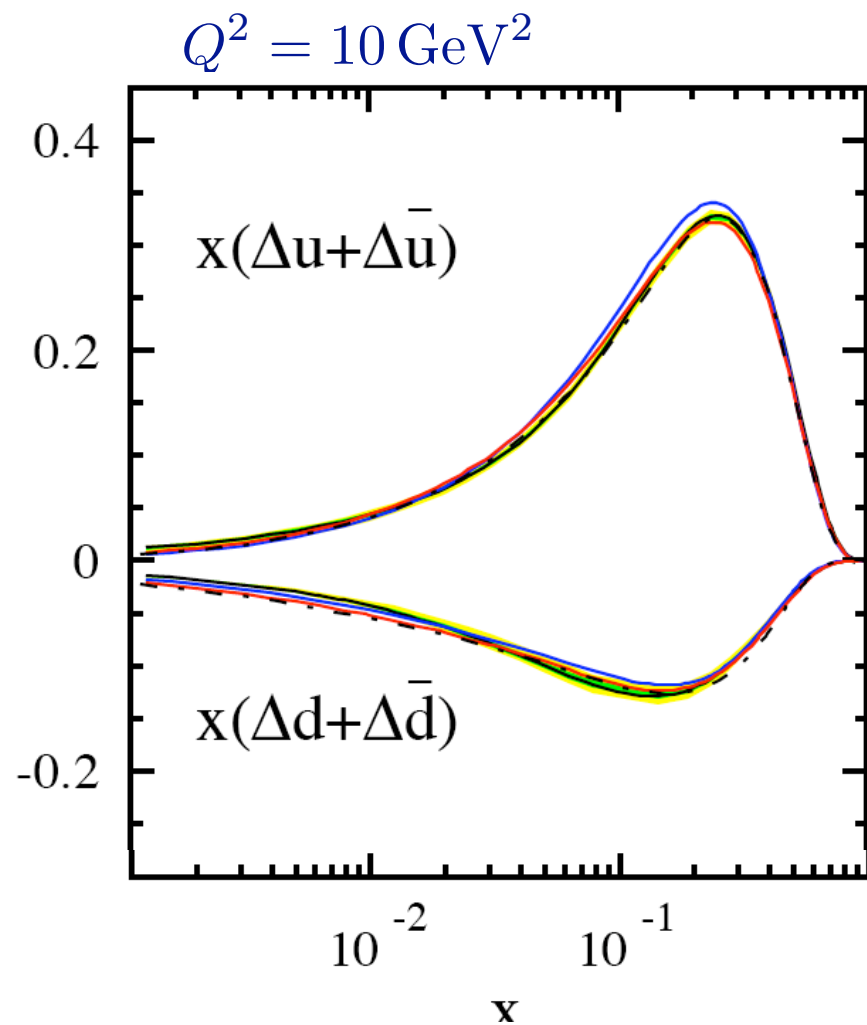


$$\chi^2/\text{pdf} = 0.86$$

Green bands corresponds to  $\Delta\chi^2 = 1$

Yellow bands corresponds to  $\Delta\chi^2/\chi^2 = 2\%$

# PDFs and “uncertainties”



$\Delta \bar{u} \sim$  positive

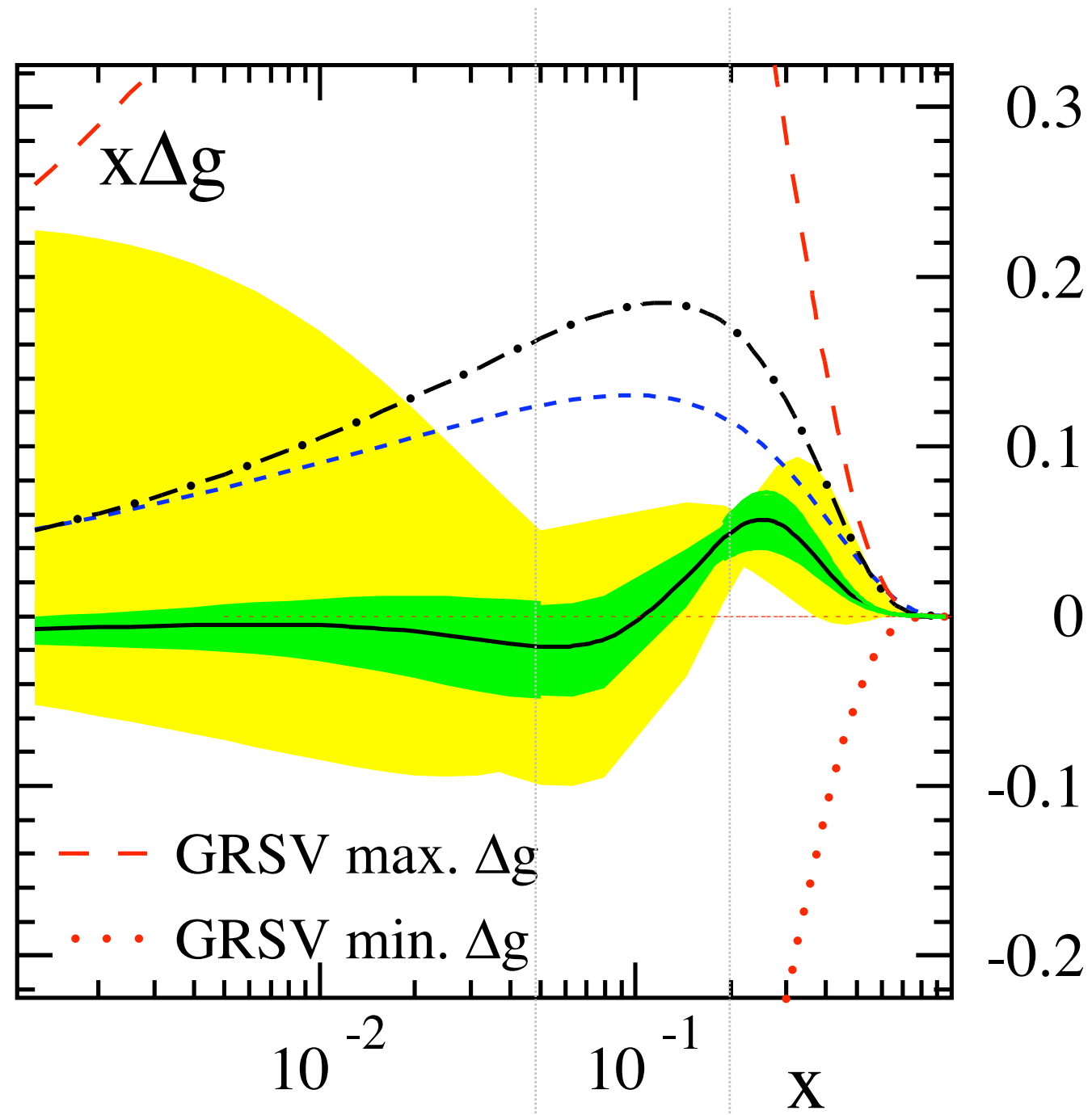
$\Delta \bar{d}$  : negative

$\Delta \bar{s}$  : **SIDIS** requires positive (**HERMES**)  
but first moment negative (**DIS**)

Robust pattern : ~~SU(3)~~ sea

SIDIS relies on DSS frag. functions **DdeF, Sassot, Stratmann (DSS)**

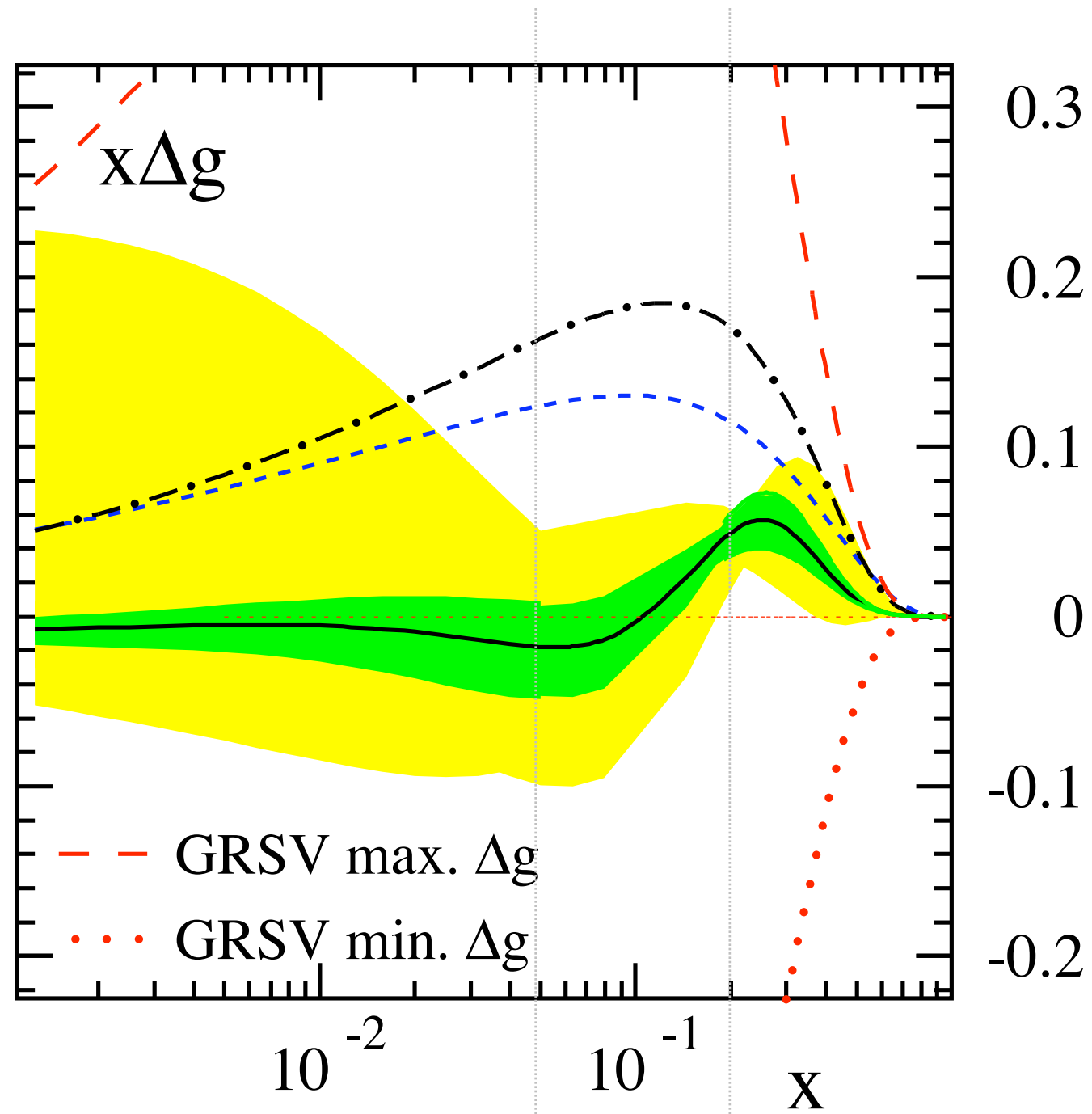
# Gluon



Best fit has a node, negative below  $x \sim 0.1$

- DSSV
- - - DNS
- · - GRSV
- DSSV  $\Delta\chi^2=1$
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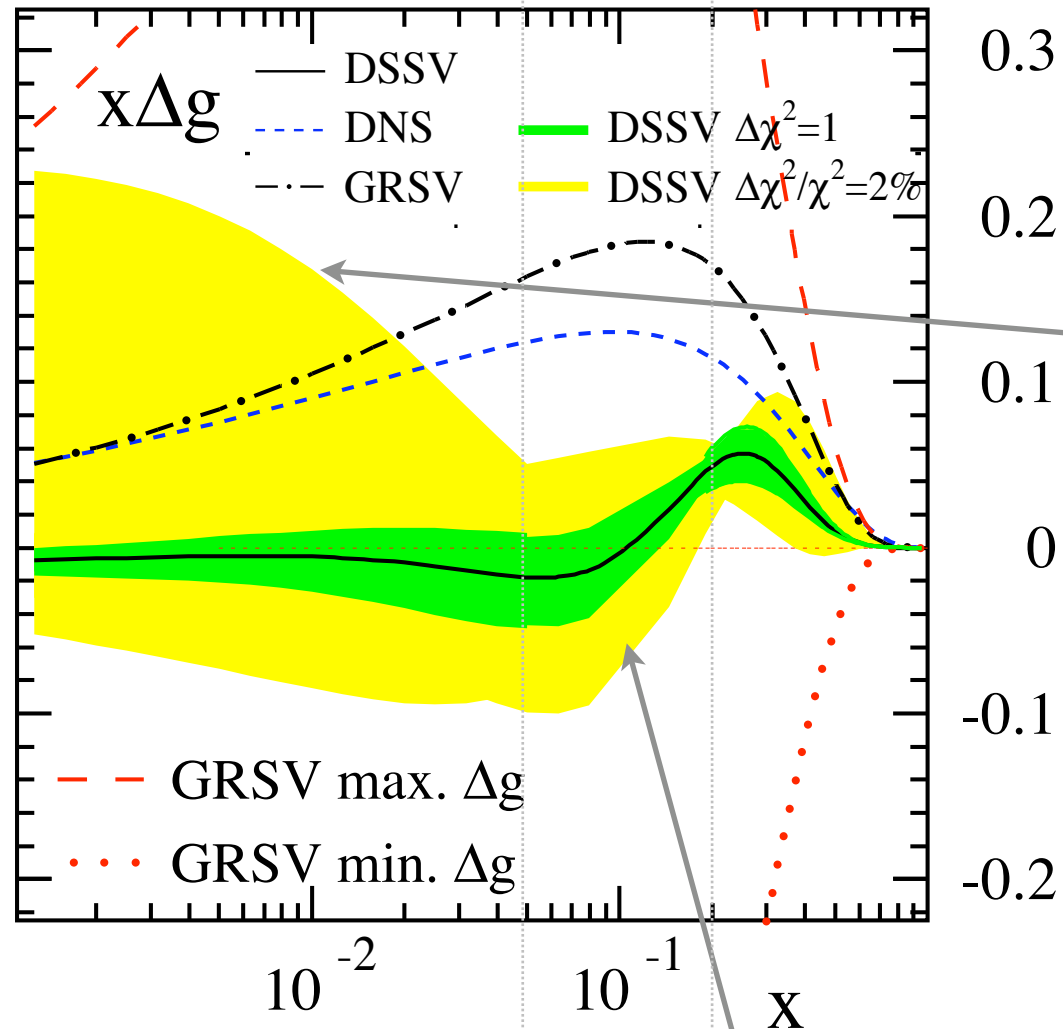


# Gluon

Moments  $Q^2 = 10 \text{ GeV}^2$

	$x_{\min} = 0$	$x_{\min} = 0.001$	
	best fit	$\Delta\chi^2 = 1$	$\Delta\chi^2/\chi^2 = 2\%$
$\Delta g$	-0.084	$0.013^{+0.106}_{-0.120}$	$0.013^{+0.702}_{-0.314}$

Best fit : small first moment ... but..



No clear statement possible for First moment :  $\sim 0$  but huge uncertainty at small  $x$

but great improvement at medium  $x$

# Sum Rule

$$\int_{x_{\min}}^1 \Delta f(x, Q^2) dx$$

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$\Delta u + \Delta \bar{u}$	0.813	0.793 $^{+0.011}_{-0.012}$	0.793 $^{+0.028}_{-0.034}$
$\Delta d + \Delta \bar{d}$	-0.458	-0.416 $^{+0.011}_{-0.009}$	-0.416 $^{+0.035}_{-0.025}$
$\Delta \bar{u}$	0.036	0.028 $^{+0.021}_{-0.020}$	0.028 $^{+0.059}_{-0.059}$
$\Delta \bar{d}$	-0.115	-0.089 $^{+0.029}_{-0.029}$	-0.089 $^{+0.090}_{-0.080}$
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$\Delta g$	-0.084	0.013 $^{+0.106}_{-0.120}$	0.013 $^{+0.702}_{-0.314}$
$\Delta \Sigma$	0.242	0.366 $^{+0.015}_{-0.018}$	0.366 $^{+0.042}_{-0.062}$

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of preliminary results, speculative ideas..

**Michel Garçon**

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Michel Garçon

What if ?  $\Delta G \sim -\frac{1}{2}\Delta\Sigma$

AdS/CFT result in strong  
coupling limit (Gao, Xiao, Hatta, Ueda)

## 'Static' Scenario?

What if ?  $\Delta G \sim -\frac{1}{2}\Delta\Sigma$  (at some  $Q^2$ )

$$\cancel{\frac{1}{2}\Delta\Sigma} + \cancel{\Delta G} + L_q + L_G = \frac{1}{2}$$

All spin by OAM

max. violation of naive expectation

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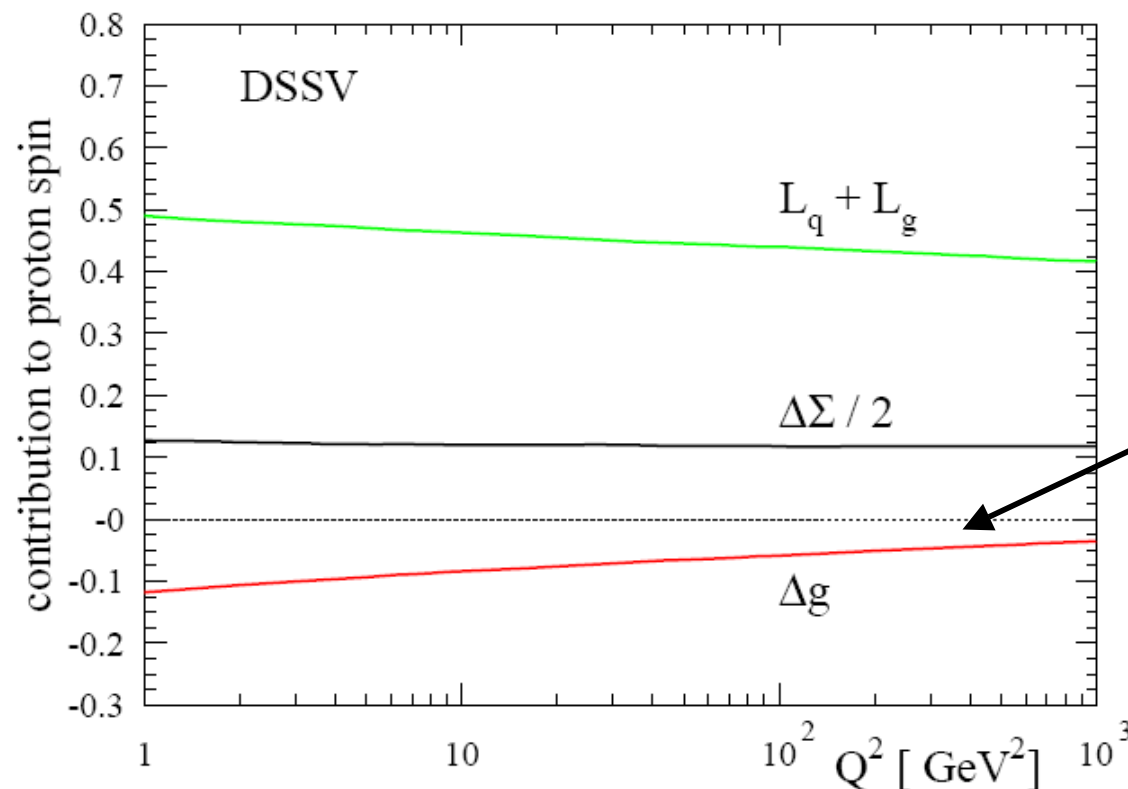
Evolution of first moment at LO

$$\frac{d}{d\ln(\mu^2)} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} 0 & 0 \\ \frac{3}{2}C_F & \frac{1}{2}\beta_0 \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}$$

$\begin{matrix} 2 & \sim 4 \end{matrix}$

← constant  
 ← constant if  $\Delta G \sim -\frac{1}{2}\Delta\Sigma$

(all  $Q^2$ )



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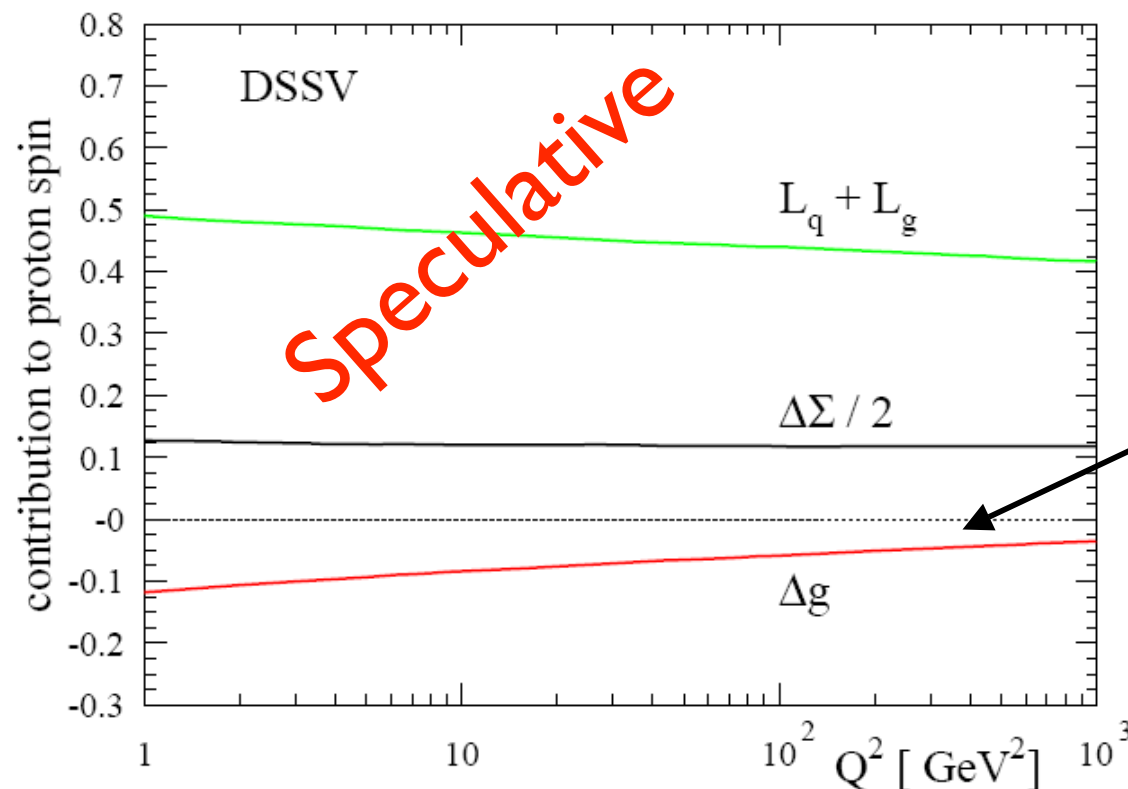
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DSSV close to static

Any nucleon model  
supports this speculation?

# Future of $\Delta G$

- TH: Understand better COMPASS observables (and keep asking for cross-sections!)
- DIS: COMPASS and JLAB-12 data (evolution)
- RHIC: more precise and at 500 GeV (smaller x)
- Less inclusive observables
  - Dijets
  - Jet + pion
  - pion + photon
- Less **inclusive** allows to perform a more detailed selection
  - Cuts to enhance sensitivity on some partonic channel
  - Slightly different scales



# Summary

- Learnt a lot about polarized pdfs and proton spin contribution
- Reasonable knowledge on quark, more work needed for antiquarks
- Gluon polarization much smaller than expected at medium x

$$Q^2 = 10 \quad \Delta G \sim 5 \quad \xrightarrow{\text{Anomaly}} \quad \Delta G \sim 1 - 2 \quad \xrightarrow{\text{DIS}} \quad \Delta G \sim 0? \quad \Delta G \sim -\frac{1}{2}\Delta\Sigma? \quad \text{Global}$$

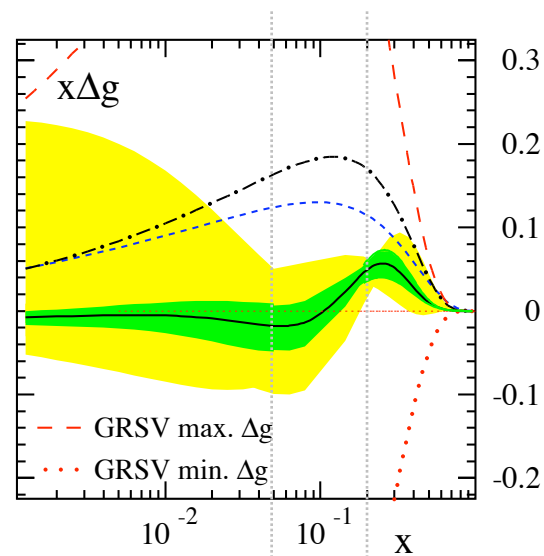
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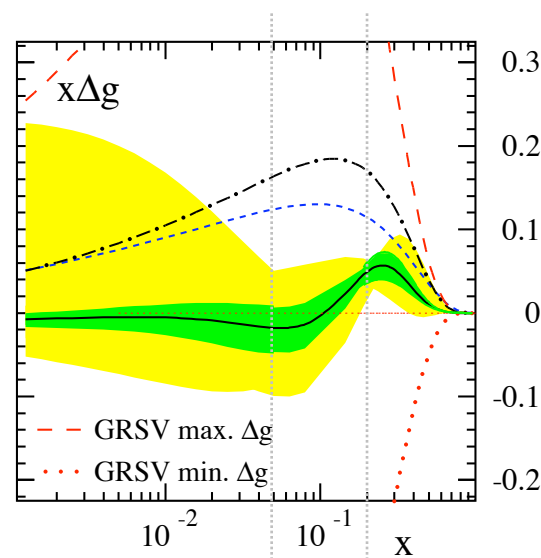
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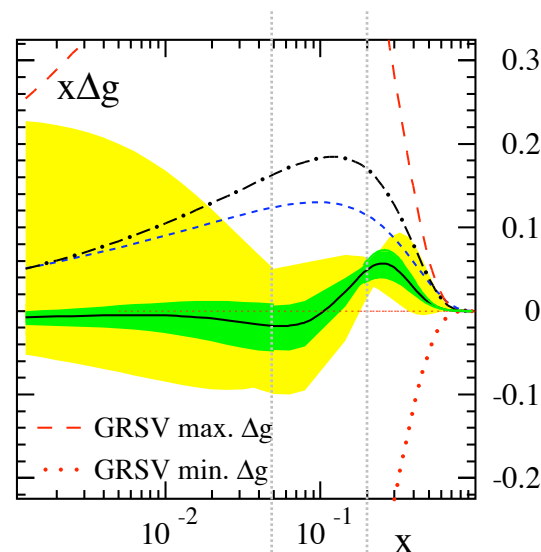
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$$\int_0^1 \Delta g(x, Q^2) dx \quad \text{Need gluon at small } x$$

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- Polarized ep collider needed to get first moment

**Thanks**



# Uncertainties



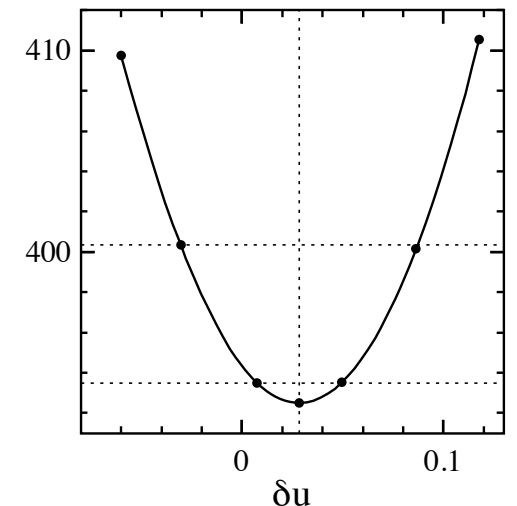
Use **Lagrange Multipliers** technique to **estimate** uncertainties (from exp. errors) on **some observables**

$$\Phi(\lambda_i, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i \mathcal{O}_i(\{a_j\})$$

See how fit deteriorates when PDFs forced to give different prediction for  $\mathcal{O}_i$

$\Delta\chi_n^2$  should be parabolic if data set can determine the observable (otherwise monotonic or flat)

$\Delta\chi_n^2$  We take a pragmatic 2% to define **uncertainty bands**

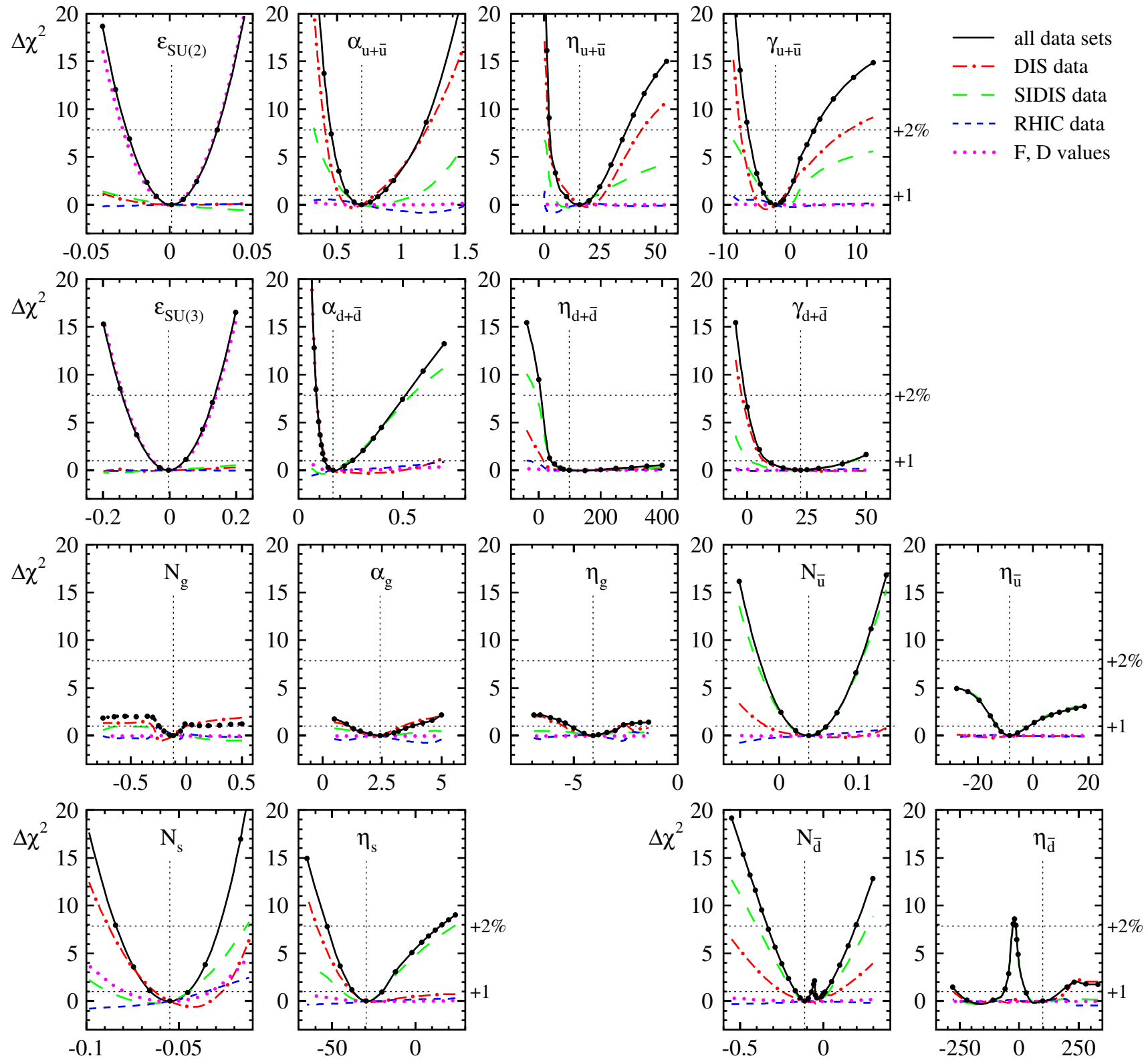


Use **Hessian Method** to **estimate** uncertainties (from exp. errors) on **pdfs**  
(J.Pumplin and CTEQ)

38 DSSV eigenvector sets to compute uncertainties on any observable  
**Unfortunately** only for  $\Delta\chi_n^2 = 1$

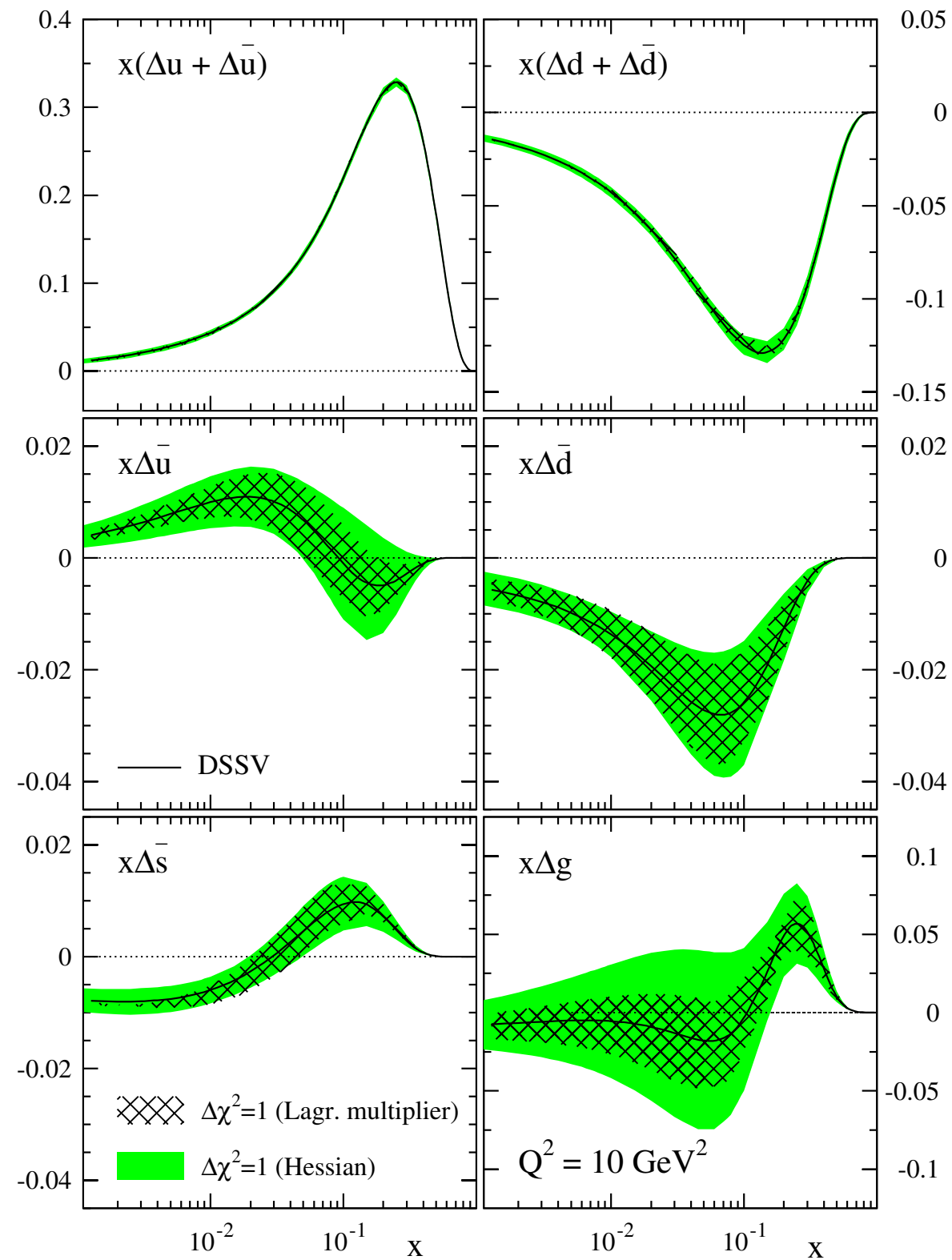
$$\Delta\mathcal{O}_i = \frac{1}{2} \left( \sum_{k=1}^{N_{\text{par}}} [\mathcal{O}_i(S_k^+) - \mathcal{O}_i(S_k^-)]^2 \right)^{1/2}$$

# Profiles



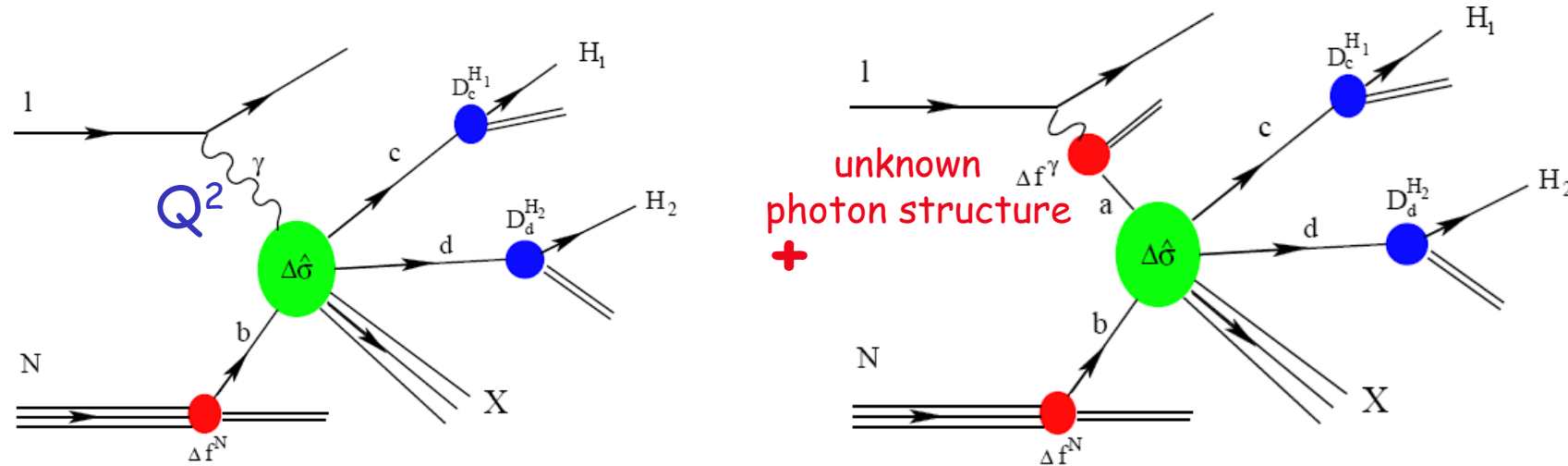


# Hessian vs Lagrange

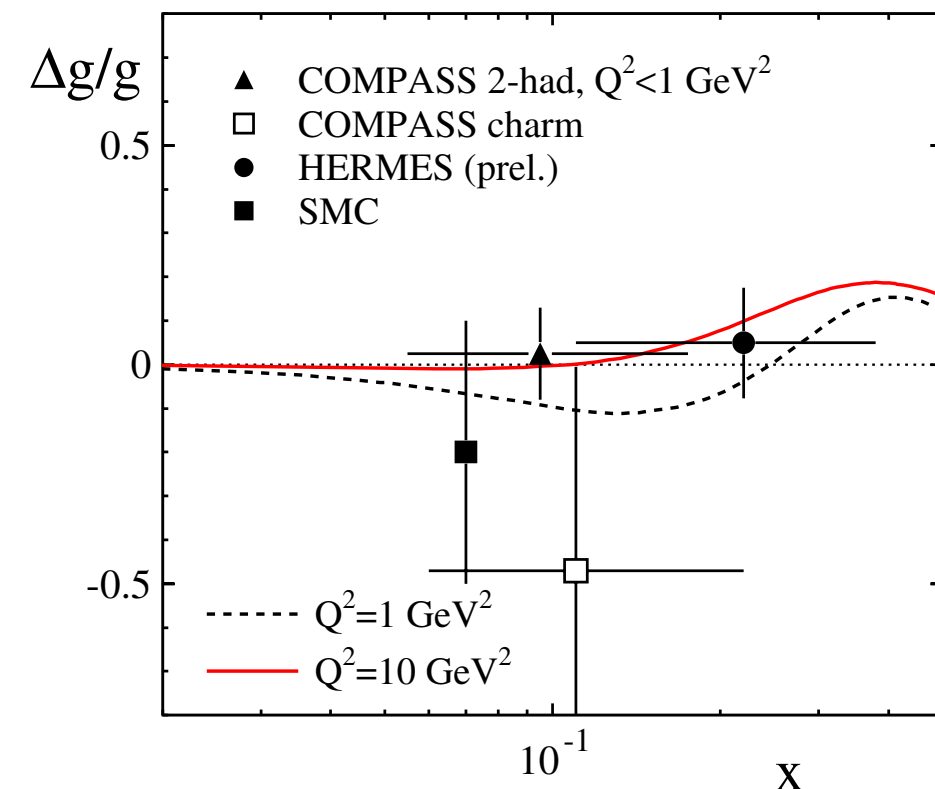


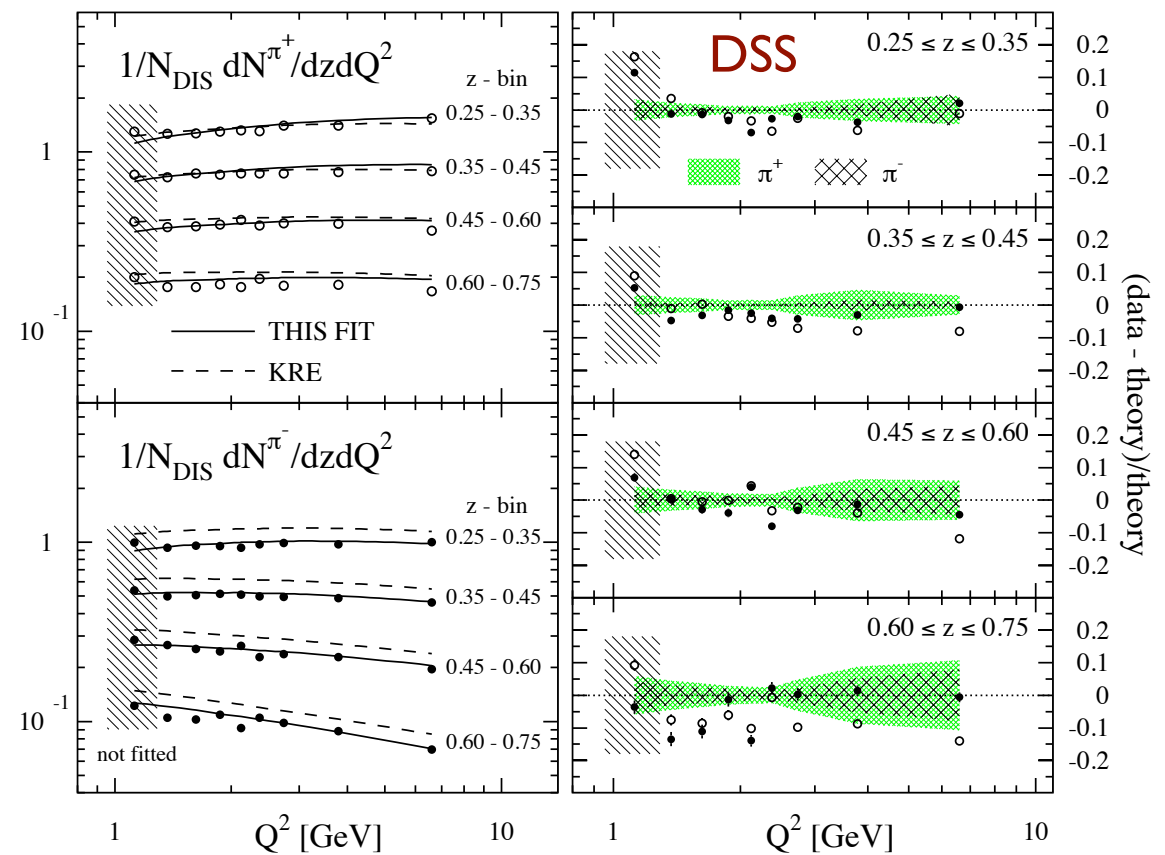
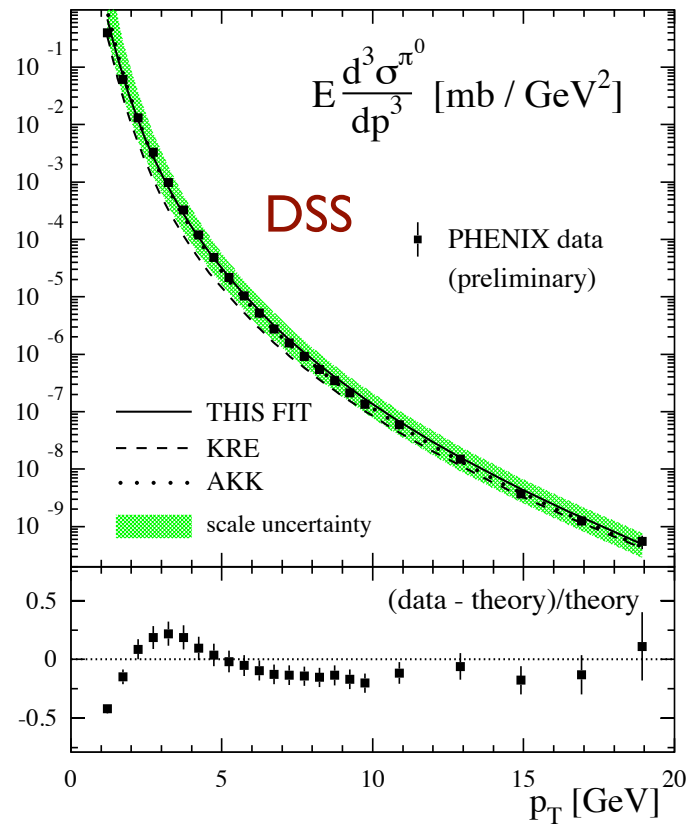
# Compass/Hermes $\Delta G/G$

Two hadron production : gluon enters at Lowest order enhanced contribution



- \* Low transverse momentum/less inclusive  $\Rightarrow$  Factorization/pQCD at risk?
- \* Photoproduction : resolved polarized photon?
- \* only lowest order analysis : if factorization works, NLO corrections expected to be large





Hermes

✓ Pion production at RHIC OK  
 $p_T^\pi \geq 1 \text{ GeV}$

✓ SIDIS OK with DSS fragmentation functions  
 DdF, R.Sassot, M.Stratmann

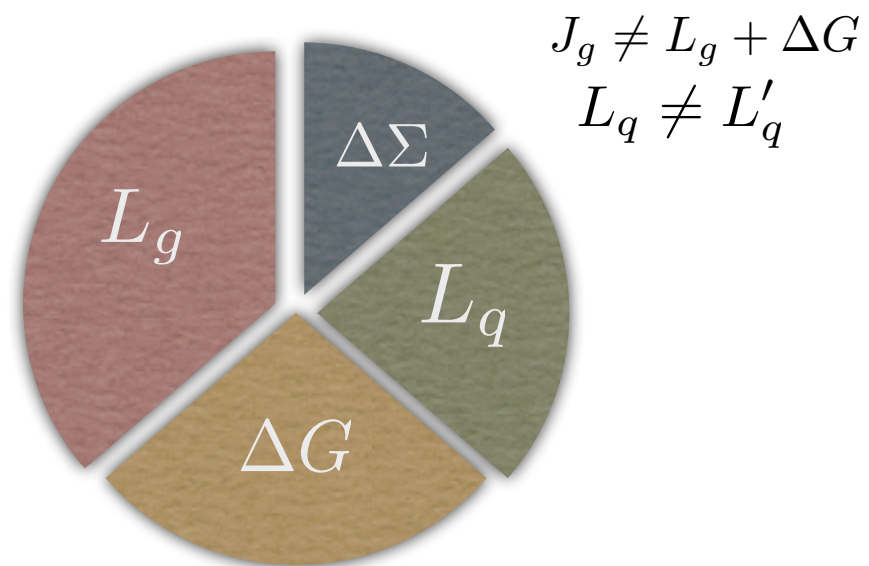
**DSS** fragmentation functions extracted from a global fit that includes RHIC and HERMES unpolarized data!

All other FF sets fail to reproduce Hermes data

# SPIN SUM RULES

Adapted from M.Burkardt

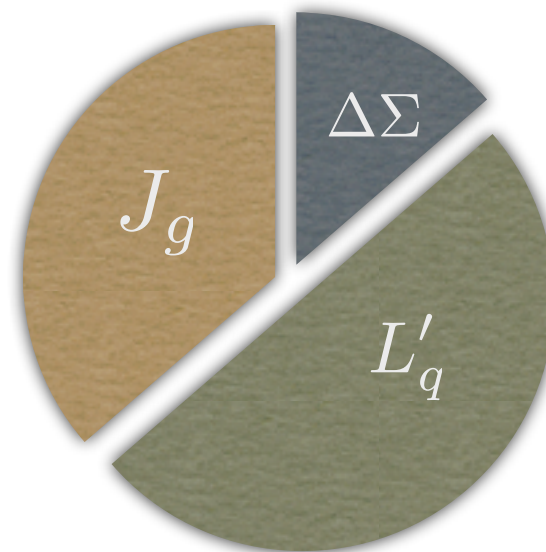
Jaffe, Manohar



Partonic interpretation  
local operator only in  
light cone gauge

$$\frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g = \frac{1}{2}$$

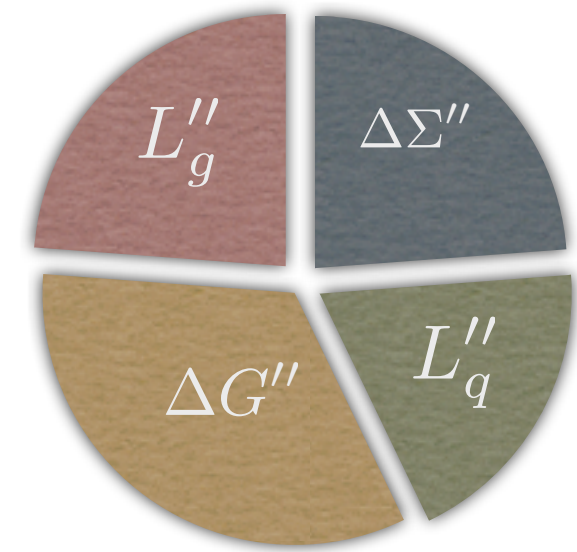
Ji



Gauge invariant  
Lattice and GPDs  
Contains interactions

$$\frac{1}{2}\Delta\Sigma + L'_q + J_g = \frac{1}{2}$$

Chen, Lu, Sun, Wang, Goldman



Gauge invariant  
physical interpretation ?  
related to new pdfs ?

$$\frac{1}{2}\Delta\Sigma'' + \Delta G'' + L''_q + L''_g = \frac{1}{2}$$

# Technical Issues and Mellin

Technical Problem: several evaluations of **evolved** PDFs and **observables**

Evolution of PDFs and NLO corrections involve convolutions: time consuming

**Mellin** space : convolutions turn into products

$$\Delta f_j^n(Q^2) \equiv \int_0^1 dx x^{n-1} \Delta f_j(x, Q^2) \qquad \int f \otimes g \rightarrow f^n \times g^n$$

**Exact** (analytic) solution for DGLAP equations and DIS and SIDIS coefficients

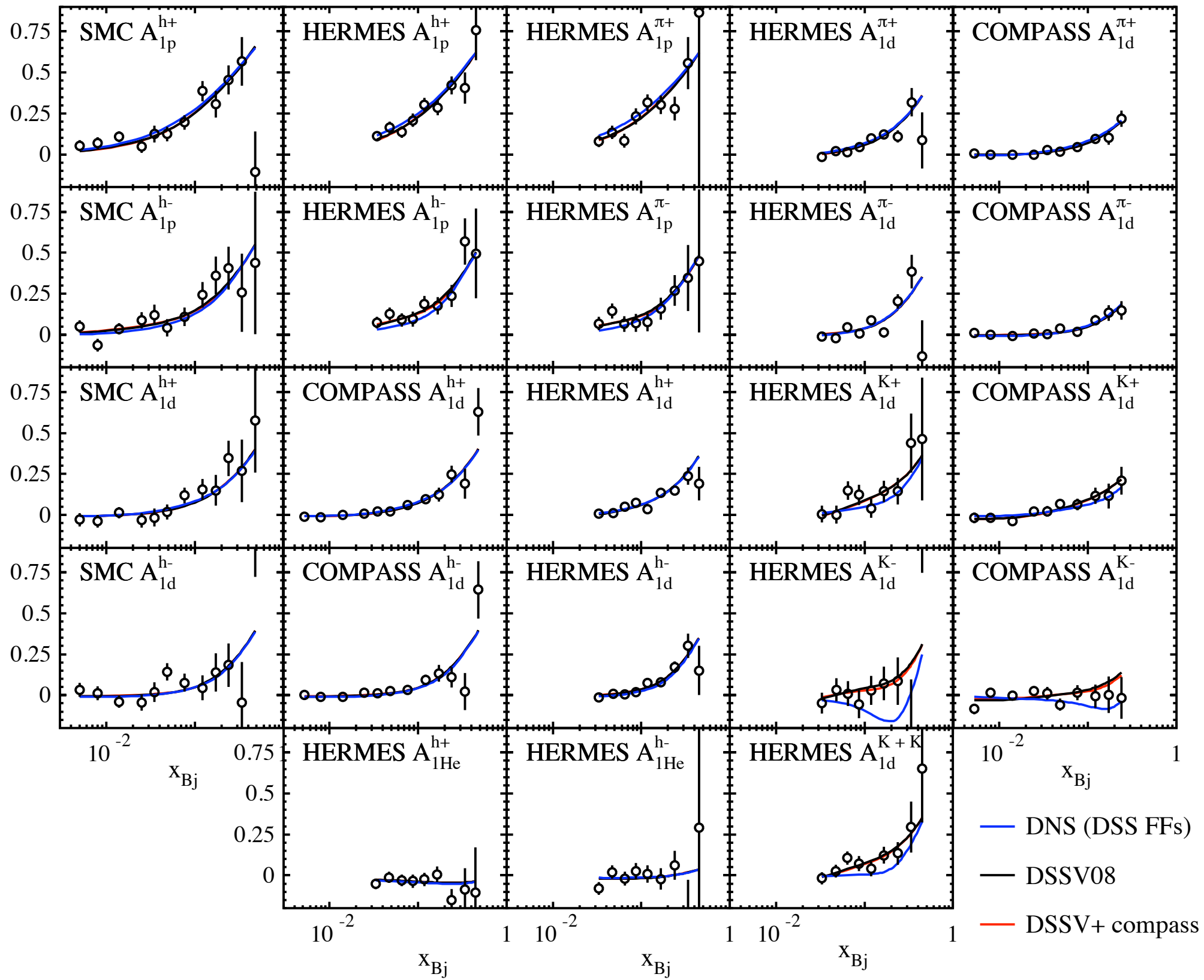
**Efficient** (fast) inversion possible

Rather more complicated for pp observables

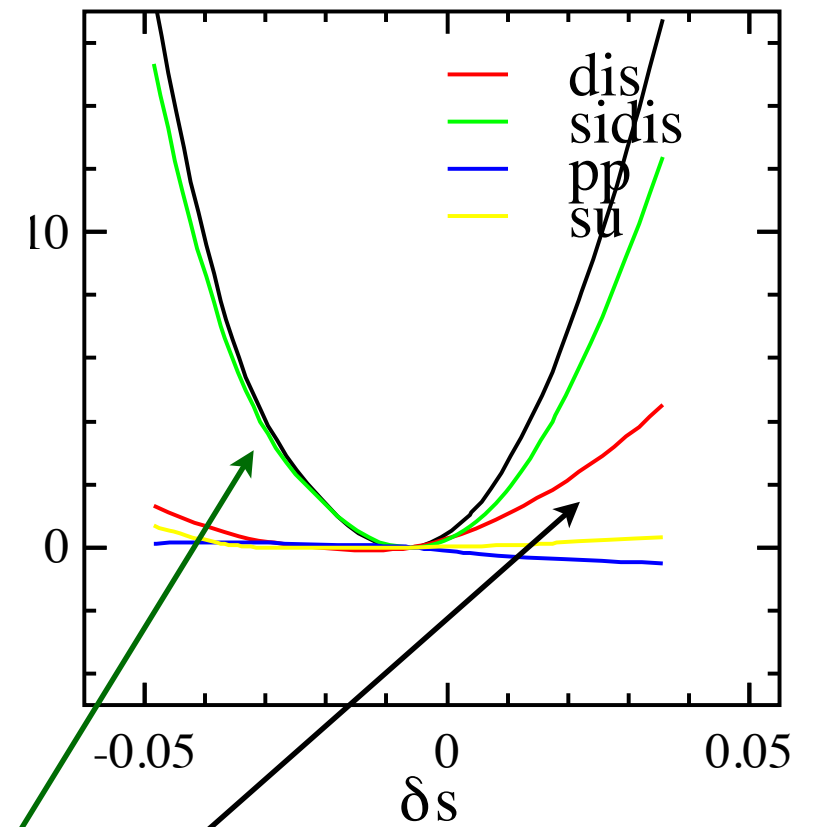
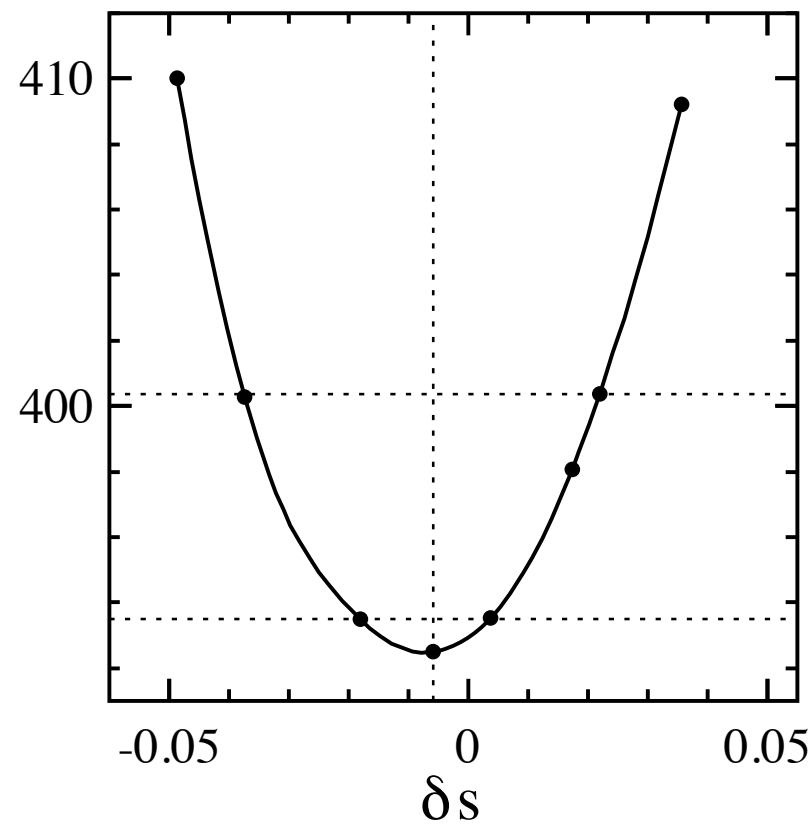
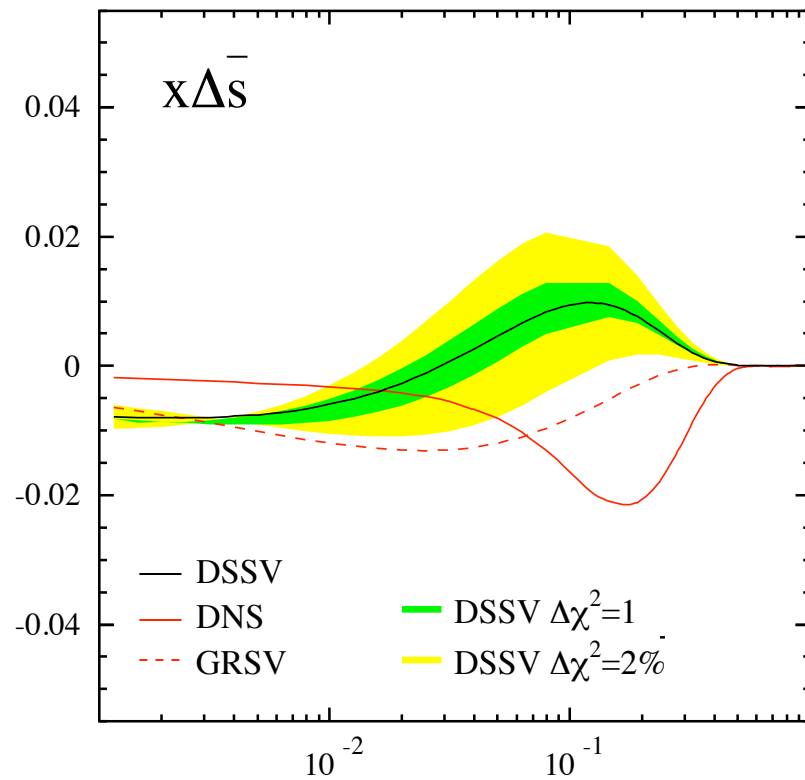
$$d\Delta\sigma = -\frac{1}{4\pi^2} \sum_{ab} \int_{C_n} dn \int_{C_m} dm \Delta f_a^n(Q^2) \Delta f_b^m(Q^2) \\ \times \underbrace{\int dx_a \int dx_b x_a^{-n} x_b^{-m} d\Delta\hat{\sigma}_{ab}(x_a, x_b, \dots)}_{\equiv \Delta\hat{\sigma}_{ab}^{n,m}} \quad \text{pre- compute } n \times m \text{ grids in complex space}$$

M.Stratmann, W.Vogelsang

New: any observable possible using sampling techniques faster evaluation ~1 day-computer  
dijets with exp. cuts



# Flavor separation still dependent of FF (unpol. Hermes multiplicities)

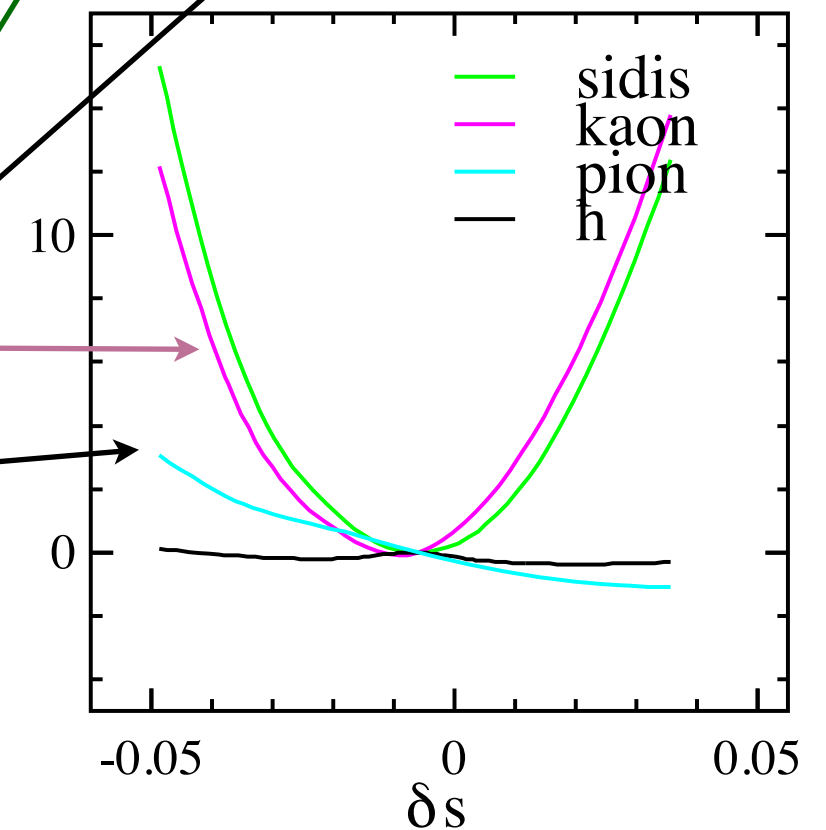


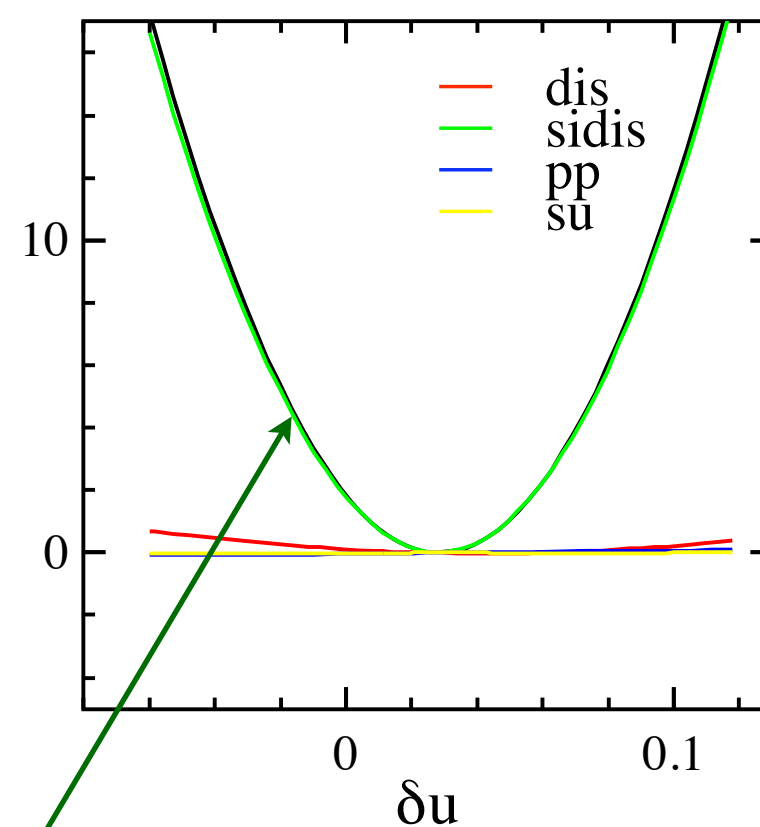
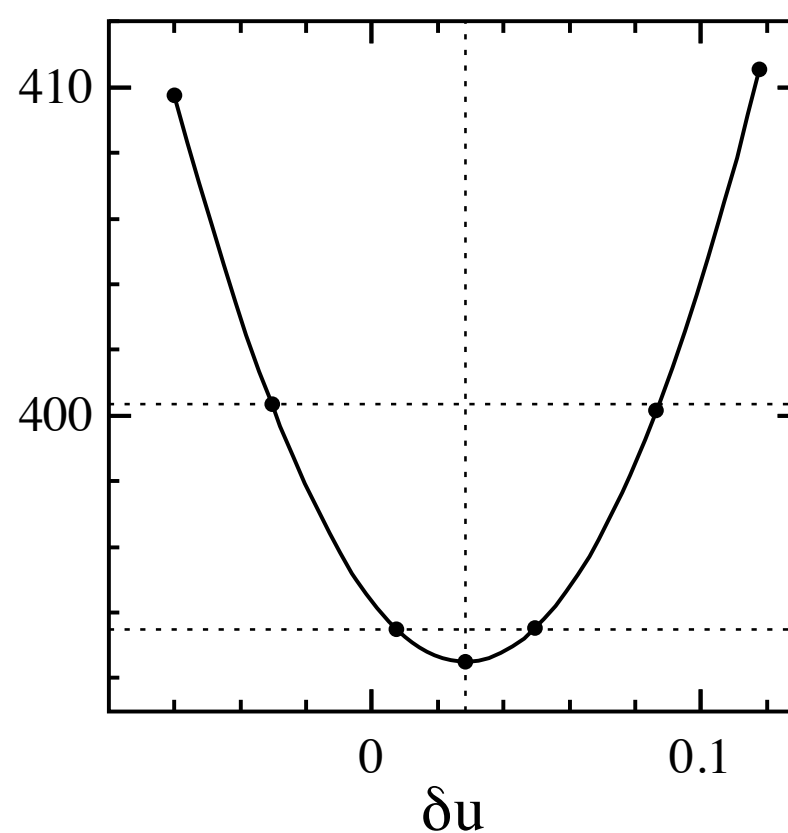
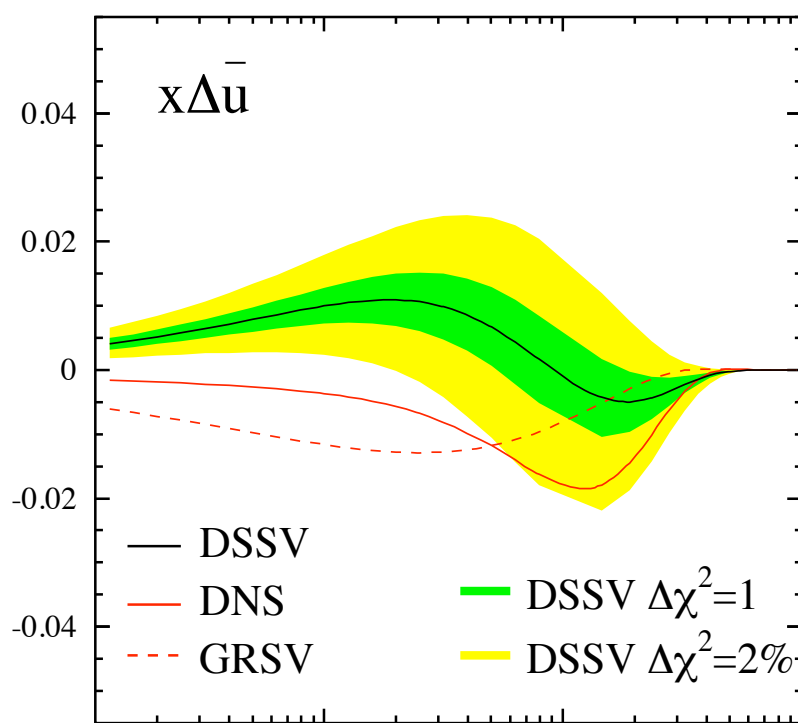
$$\delta\bar{s} \equiv \int_{0.001}^1 \Delta\bar{s} dx$$

sidis clearly dominate

kaons dominate

pions + dis  
confabulation





sidis clearly dominates  
charged hadrons dominance  
pions agree ...

